GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION
SPONSORED PROJECT INITIATION

Date: 12/11/80

Project Title: Computational Methods in Advanced Stress and Durability Analysis of Aircraft Structures

Project No: E-20-674

Project Director: Dr. S. Atluri

Sponsor: Air Force Office of Scientific Research, Bolling AFB, D.C.

Agreement Period: From November 15, 1980 Until ~~November 14, 1981~~ 2/28/82

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Defense Priority Rating: None

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SPONSORED PROJECT TERMINATION SHEET

Date 3/22/82

Project Title: Computational Methods in Advanced Stress and Durability Analysis

Project No: E-20-674

Project Director: Dr. S. Atluri

Sponsor: AFOSR, Bolling AFB, DC

Effective Termination Date: 12/31/81 (1st year)

Clearance of Accounting Charges: 12/31/81 (1st year)

Grant/Contract Closeout Actions Remaining:

- [ ] Final Invoice and Closing Documents
- [x] ~~Final~~ Interim Fiscal Report
- [ ] ~~Final~~ Interim Report of Inventions
- [ ] Govt. Property Inventory & Related Certificate
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Continued by E-20-671

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Interim Scientific Report

on

AFOSR Grant 81-0057

titled

Computational Methods in Advanced Stress and Durability Analysis of Aircraft Structures

Submitted to
Air Force Office of Scientific Research
Building 410, Bolling Air Force Base
District of Columbia 20332
Attn: Dr. Anthony Amos

Prepared by
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Introduction:

In the following pages, the scientific progress made during the period 11/15/80 to 12/31/81, under AFOSR grant 81-0057 is summarized. Following a summary of the technical accomplishments, a list of research papers written in archival literature based on the work accomplished, is given grant.

S.N. Atluri of the Center for the Advancement of Computational Mechanics was the principal investigator on the subject.

Description of Scientific Accomplishments:

General variational theorems for the rate-problems of rate-independent finite strain inelasticity in terms of the appropriate rates of the first and second Piola-Kirchhoff stress tensors, the symmetrized Biot-Lure' stress tensor, and their conjugate measures of strain rate have been derived. Certain new rate-complementary-energy principles, involving the rate of spin and the rate of symmetrized Biot-Lure' stress tensor as variables, for finite strain analysis of rate-sensitive materials such as those exhibiting elasto/viscoplastic/creep behavior, have been discovered. This work is documented in [1].

Several new concepts involving the use of stress functions and asymptotic solutions to the governing differential equations for formulating finite element procedures for the analysis of linear and nonlinear problems of mechanics of solids as well as fluids have been explored, and successfully applied. These have been documented in [2].

Stress-based finite elements, and attendant computer programs, were developed for applications to problems of large-deformation viscoplasticity and creep. In their formulation, the equations of compatibility and angular momentum balance are descritized, and the surface velocities act as the Lagrange multipliers by which inter-element traction reciprocity as well as traction boundary conditions are enforced. The basis of the development is the new complementary energy rate principle reported in [1]. The above finite element method yields an initial value problem for deformation and stress. Suitable time-integration operators have been explored. The results are documented in [3].

Computational techniques, which preserve the objectivity of incremental constitutive relations, for finite elastic or inelastic materials, for finite time steps, during which the material elements may undergo finite rotations, have been developed and reported in [4]. These techniques have been found to be extremely useful in finite deformation (finite rotations, finite strain) computational analyses of two as well as three dimensional solids and structures.

A new procedure for the analysis of finite deformations, finite rotations, buckling and post-buckling behavior of arbitrary shaped shells of elastic or inelastic materials is being developed. Polar-decomposition of the shell deformation into rigid rotation and pure stretch is considered. Initial phase of development of shell finite elements based on a complementary energy principle, wherin unsymmetrical nominal stresses and rigid rotations are treated as variables, has been completed. The initial results are presented in [5].

Considerable research efforts have been expended to develop numerical techniques for problems with constraint. Such problems are typified by material behavior such as incompressibility. Examples include incompressible elastic materials (as in solid propellant rocket grains) as well as incompressible Newtonian fluid. The mathematical problems as well as numerical treatment are essentially similar in both cases. There has been a considerable amount of work repeated in recent and current literature concerning the use of the so-called reduced-integration-penalty methods to treat such problems. However, theoretical investigations by Oden and others have shown that these methods are potentially unstable, numerically. Thus, as an alternative, research was directed at the use of hybrid and mixed/hybrid methods in the treatment of problems of incompressibility. A hybrid finite element scheme, based on assumed deviatoric continuum stresses in each element and continuous deformation field at the element boundary has been developed. The deviatoric stress and the hydrostatic pressure field are subject, a priori, to the constraints of balance of momenta. .The advantages of the developed method, and its versatility has been demonstrated through several test cases and documanted in [6,8].The stability and convergence properties of the present method, and the so-called Ladyzhenskaya-Brezzi conditions for the stability, have been analyzed and reported in [7].

A new complementary energy-based finite element method, and attendant computer coding, has been developed for the analysis of plates undergoing large deformations and instability. In this process a new and fundamentally novel plate theory which treats large rotations as fundamental variables has been developed. The kinematical treatment in this theory makes a fundamental departure from the classical theories of Kirchhoff-Love, and Reissner. This new theory enabled an efficient and simple calculation of large-deformation and post-buckling behavior of plates. The essentials of the theory and several representative numerical results have been reported in [9].

"Special hole-elements" to enable an efficient and accurate analysis of stress concentration around through-the-thickness holes in angle-ply laminates have been developed. In these "hole-elements", the analytical asymptotic solutions for the three-dimensional, equilibrated, stress-state near the hole, are embedded. The fully 3-D stress-state in the laminate is accounted for, and the interlaminar traction reciprocity is satisfied a priori, while the interelement traction reciprocity is satisfied a posteriori. In addition, a simple two-dimensional method of estimation of stress-concentration around holes in laminates has also been developed. This simple estimation method is of considerable advantage in design applications. The details of these developments and several numerical solutions are documented in [10, 11].

The essential underlying ideas of mechanics and mathematics used in the development of above computational methods have been documented in [13, 14].

A major, comprehensive, and fundamental study of the subject of energetics and path-independent integrals in fracture mechanics was undertaken. Path-independent integrals, of relevance in the presence of cracks and crack-like defects, in two- and three-dimensional elastic and inelastic solids were considered. The hypothesized material properties include: (i) finite and infinitesimal elasticity, (ii) rate-independent incremental flow theory of plasticity, and (iii) rate-sensitive behavior including elasto-viscoplasticity and creep. In each case, finite deformations were considered, along with the effects of body forces, material acceleration, and arbitrary traction/displacement conditions on the crack-face. Also the physical interpretations of each of the integrals either in terms of crack-tip energy release rates or simply energy-rate differences in two comparison cracked-bodies were explored. Several major and fundamental discrepencies between the obtained results in the present work and those currently considered well established in literature were pointed out and discussed. The results are summarized in a major paper [14].

In Ref. [14] a new incremental path-independent vector ($\Delta \underline{T}$) integral was discovered, which possesses the following fundamental properties that distinguish it from other parameters currently used in literature: it is (i) valid for arbitrary constitutive properties of the material such as rate-independent incremental elasto-plasticity, or rate-sensitive inelasticity such as viscoplasticity and creep, (ii) valid for quasi-statically or dynamically propagating cracks in ductile materials, (iii) valid for mixed mode problems, (iv) valid in the presence of arbitrary loading histories that involve loading and unloading, (v) provides a unified basis for both initiation as well as propagation criteria, (vi) valid for arbitrary crack-face conditions such as prescribed tractions and/or displacements, (vii) valid in the presence of arbitrary body forces whether due to mechanical action, or initial strains such as thermal strains, (viii) has the clear physical meaning that it is the energy-rate difference between two cracked bodies of identical shape, but differing in crack-lengths by an infinitesimal amount, ΔC, both bodies being subjected to identical load histories (including loading/unloading), and (ix) has been derived from an energy-conservation law valid for arbitrary materials with incremental constitutive laws between an objective stress rate and an objective strain rate.

The validity of the above new parameter ($\Delta \underline{T}$) has been verified in the case of creep crack growth, as may occur in structural components of the hot section of an aircraft jet engine [15, 16]. Work is underway concerning its verification in the area of elastic-plastic fracture mechanics.

In this area of fracture and damage of composite materials, a simple method of estimation of stress-intensity factors for through-cracks in angle-ply laminates is developed. Savins elasticity solution for an elliptical hole in a two-dimensional infinite plate is used as a basic solution for the stress-distribution in each ply of the laminate. Several results for carefully chosen test cases were obtained and compared with previously obtained solutions using three-dimensional finite elements. Considering the good accuracy and the simplicity of the present estimation method, it gives the designer of composites a useful tool for estimating stress concentrations/intensifications due to the presence of a hole or crack. This work is documented in [17].

A three dimensional analysis of cracks in adhesively bonded laminates, with debonding around the crack, has been conducted. Effects of the following paramatric variations were studied: (i) relative thickness of adhesives, (ii) relative material properties of adhesives and adherends, (iii) ratio of flow length, to laminate characteristic dimensions, and (iv) shape of the debonded region. These results effectively serve as bench marks for any further-simplified analysis and/or experimental work. These have been reported in [18].

Studies were conducted into the derivation of singular finite elements which can generate any prescribed singularity in the interpolated function derivative. Both C° and C' type triangular and quadrilateral elements were considered. Mathematical theorems concerning the singularity and strain energy of these elements were established and verified. These elements are useful in program design due to their inherent simplicity, compatibility, and ease of implementation. These are reported in [19,20].

A major development was undertaken to devise a cost-effective 3-D alternating method, for easy use in the design environments of aerospace industry, for an accurate estimation of stress-intensity factors for surface flaws. Typical of such examples are: corner cracks near fastener holes in panels, corner cracks near holes in attachment lugs in aircraft landing gears, etc. Until recently such analyses were prohibitively expensive and the industrial practice appeared to be to conduct simplified two-dimensional analyses, which, recent experience shows, were not always conservative. The presently developed method is expected to make such analyses computationally and cost-wise feasible as a routine. The key ingredients in the presently developed method are: (i) analytical solution for an embedded elliptical flaw in an infinite medium, the crack-faces being subjected to arbitrary shear as well as normal tractions, and (ii) a crude finite element analysis of the uncracked solid to obtain an estimate of stresses at the prospective crack location and (iii) an "alternating" method which uses, iteratively, to erase the stresses obtained in step (ii), making use of the analytical solution (i). In this research, steps (i) and (iii) proved to be rather ambitious undertakings, but successfully accomplished. Results obtained so far indicate that the present method is at least 20 times less expensive to use than all the previous methods including the ones developed by the writer in earlier years. These results are documented in [21,22] and it is hoped that the method would have a major impact on durability and integrity studies of aircraft design.

Also, a preliminary effort was made at comparing our analytical work with independent experimental work concerning damage tolerant design of panels with surface flaws, and a study of lower bound flaw shapes. This study reported in [23] is encouraging and will be pursued.

Finally, to study crack arrest in stiffened plates, efforts were expended in analyzing dynamic crack propagation and arrest. Studies of the type of both: (i) "generation" i.e., determination of dynamic stress-intensity factor variation with time for a specified crack-propagation history, as well as (ii) "prediction" i.e., determination of crack-propagation history for specified dynamic fracture toughness versus crack-velocity relationships were performed. These studies were conducted by using a transient finite element method wherein the propagating stress-singularities near the propagating crack-tip have been accounted for. These studies convincingly demonstrated that a crack arrest evaluation methodology is well within reach, and is being pursued. The results so far are documented in [24,25]. Finally, to analyze cracks in plates under bending loads, special finite elements were developed and reported in [26].

Doctoral Disertations:

The following Ph.D. disertations were written under AFOSR support.

(1) Dr. Randy Stonesifer, Thesis: "Fast Fracture and Creep Crack Growth: Moving Singularity Finite Element Analysis" (Dec. '81).

(2) Mr. Ken Reed, Thesis: "Analyses of Large Quasistatic Deformations of Inelastic Solids by a New Stress Based Finite Element Method" (Expected to graduate, April '82).

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